Transmission of a Laser Beam Through Anisotropic Scattering Media

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Experimental and theoretical two-dimensional results are compared for laser beam transmission through a multiple scattering medium. An azimuthally symmetric He-Ne laser beam with a Gaussian radial distribution was incident normal to a scattering media consisting of double distilled water and uniform-sized spherical latex particles 0.481 μm in diameter. The transmitted radiation was measured as a function of optical radius from the beam and optical thickness of the scattering medium. It is shown that 1) anisotropic scattering experimental data correlates with scattering theory for optical thicknesses of 2.0–10.0 when effective optical properties are used and 2) changing the radius-to-depth ratio of the scattering media between 0.33 and 1 does not effect the transmission for the range of optical thicknesses mentioned above.

Nomenclature

c	= effective scattering coefficient, $6C_{sca}/(\pi d^3)$
C_{sca}	= scattering cross section
d	= scattering particle diameter
g	= asymmetry factor
I	= intensity incident on the media
I_i	= magnitude of laser beam intensity
$\bar{I}_0(x)$	= zeroth-order modified Bessel function of the first kind
I_{exp}	= measured intensity leaving medium normal to the surface
I_d	= theoretical diffuse intensity
$I_N(\tau_r, \tau_0)$	= theoretically predicted intensity leaving the media normal to the surface
L	= depth of water
n	= index of refraction of water at λ_0
N	= particle number density
P_d	= detected power
P_i	= incident laser power
P	= scattering phase function
q	= radiative flux
r	= radial distance from center of the laser beam
r_0	= effective laser beam radius
R	= scattering medium radius
R_0	= detector probe radius
V	= voltage
z	= distance into scattering medium (normal to surface)
$\delta(x)$	= Dirac delta function
θ	= polar angle used to specify the direction of intensity
$\bar{\theta}$	= effective acceptance angle of detector probe
γ	= angle between incident and scattered radiation
κ	= absorption coefficient of scattering medium
λ_0	= laser beam wavelength in air, 0.6328 μm
μ	= $\cos\theta$
τ_r	= radial optical coordinate
τ_0	= optical depth of scattering medium
τ_{r_0}	= optical radius of incident beam
ϕ	= azimuthal angle, used to specify the direction of the intensity measured between the projection of the intensity on the r-ψ plane and the r axis
ψ	= azimuthal angle, used to specify the location of a point in the medium, measured around the z state
$\Delta\Omega$	= incremental solid angle
ω	= albedo
η	= particle volume concentration, $N\pi d^3/6$

Superscripts

( )*	= effective quantity accounting for anisotropic scattering and albedo

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Introduction

ANISOTROPIC, multiple scattering occurs in a wide variety of physical situations, such as underwater visibility,[1-3] atmospheric transmission,[4] blood and skin tissue analysis,[5,6] thermal insulation,[7] rocket exhausts,[8,9] and combustion systems.[10] Turbine blade wear has been measured using laser light scattering by Wyler and Desai.[11] They developed a radiative transfer equation to predict attenuation of light by wet steam. They also used laser scattering techniques to measure the wetness of the steam in a multistage steam turbine. These references represent a few of the applications of anisotropic, radiative scattering in physical situations. Most of the above applications involve one-dimensional radiative transfer.

One-Dimensional Transmission

Several one-dimensional radiative transfer investigations for anisotropic scattering media are available in the literature. Scott et al.[12] experimentally investigated the transmission of light through a multiple-scattering medium of finite thickness. They achieved reasonable agreement with the theoretical results of Chu and Churchill.[13] A comparison of experimental and theoretical multiple scattering has been carried out by Hottel et al.[14-16] and Sarofim et al.[17,18] These investigations were primarily concerned with bidirectional reflectance and transmittance from a layer of water containing spherical latex particles of uniform size. The investigators considered the effects of multiple scatter, absorption, change in the index of refraction at the boundaries, and polarization. Bravo-Zhivotovskiy et al.[19] compared experimental and theoretical results for transmission using the small-angle approximation. Brewster and Tien[20] compared theoretical and experimental results for both transmission and reflection in packed fluidized beds. They were concerned with determining the boundary between independent and dependent scattering. The interparticle spacing to wavelength ratio was shown to be the critical parameter to determine the importance of dependent scattering. In general, all the one-dimensional theoretical and experimental comparisons reported in the literature show good agreement.

Two-Dimensional Transmission

Two-dimensional radiative transfer in anisotropic scattering media has not been studied extensively. Some reflection studies are available. Look et al.[21,22] investigated reflection from a two-dimensional, multiple scattering medium using paint and also uniform-sized latex particles as scattering centers. They obtained good agreement between their theoretical solutions and experimental results for back scattered intensity. Nelson et al.[23] extended the results of Look et al.[22] to the optically thick radius region. They included the effects of finite optical depth and absorption. Modest and Tabanfar[24] compared the exact solutions for reflected radiation developed by Crosbie and Dougherty[25] with a differential approximation for absorbing, emitting, and anisotropic scattering media. They compared exact and differential approximation results for $r/r_0 = 1$ and ∞ for strong forward and strong backward scattering cases.

Ishimaru et al.[26] considered two-dimensional transmission through multiple scattering media. They compared theoretical and experimental results for various optical depths of the medium near the optically thick limit. They were concerned with experimentally verifying the range of validity for diffusion theory. They showed that diffusion theory holds for optical depths greater than unity for Rayleigh scattering and greater than 20 for Mie scattering when the scattering particles have albedos close to unity. However, when the albedo is small, diffusion theory may not be valid at all. Yuen and Dunaway[27] used a successive approximation numerical procedure to determine the scattering correction to the Beer-Lambert law for transmittance between a finite source area and an infinitesimal detecting area in a two-dimensional, rectangular absorbing and scattering medium. They considered isotropic scattering and found the magnitude of the scattering correction to be most important at intermediate optical thicknesses.

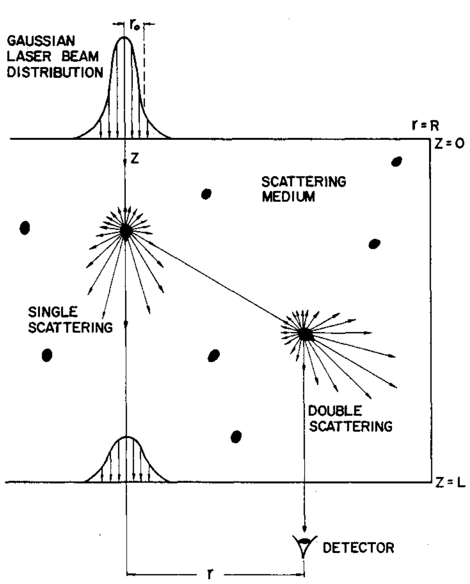

Fig. 1 Schematic of laser beam transmission through a scattering medium.

This paper presents results of a coordinated experimental and theoretical study of two-dimensional, multiple scattering radiative transfer. It was conducted to verify experimentally the theoretical transmission solutions developed by Crosbie and Dougherty[25] and Crosbie.[28] A laser beam of finite cross-sectional area is directed into a scattering medium normal to the upper surface, as shown in Fig. 1. As the beam propagates through the medium, photons are scattered out of the beam, causing it to disperse. The problem is two-dimensional, because the radiation in the medium depends on the distance along the beam and the radial distance from the beam.

The radiation transmitted through the scattering medium in the normal direction is measured as a function of radial distance from the incident laser beam. Multiple scattering occurs during the process. A photon, originally in the laser beam, must be scattered out of the beam in the radial direction and then scattered in the direction of the laser beam, so that it can exit the medium in the normal direction. The direction in which a photon is deflected during a scattering event depends on the scattering phase function. Results are presented for particles with well-characterized scattering phase functions that scatter anisotropically.

Theoretical Analysis

The theoretical development is based on the following assumptions: 1) steady state, 2) coherent scattering, 3) negligible interference and polarization effects, 4) homogeneous medium, 5) no emission, 6) no refractive index changes across all scattering medium boundaries, and 7) a two-dimensional, finite-depth scattering medium.

Incident Intensity

The incident intensity is collimated and normal to the surface of the media. It can be expressed as

$$I(\tau_r,0,\mu,\phi) = I_i\delta(\mu-1)\delta(\phi-0)\exp[-(\tau_r/\tau_{r_0})^2] \tag{1}$$

in which

$$\tau_r = (NC_{\text{sca}}+\kappa)r, \text{ and } \tau_{r_0} = (NC_{\text{sca}}+\kappa)r_0 \tag{2}$$

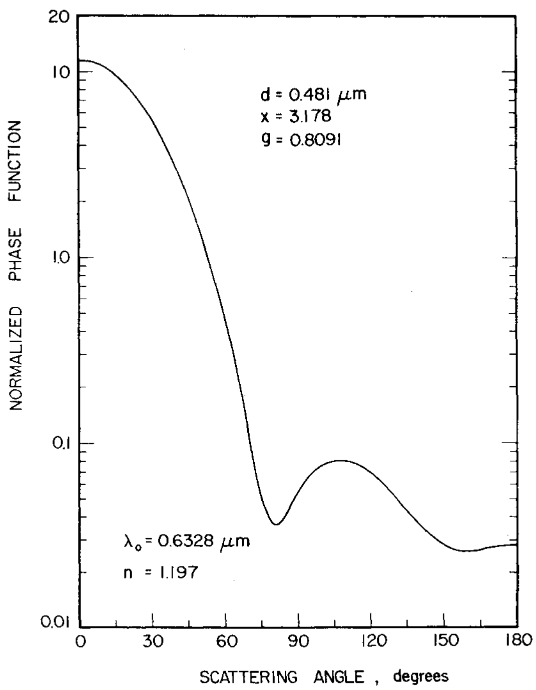

Fig. 2 Normalized theoretical phase function vs scattering angle for 0.481 μm diameter latex spheres.

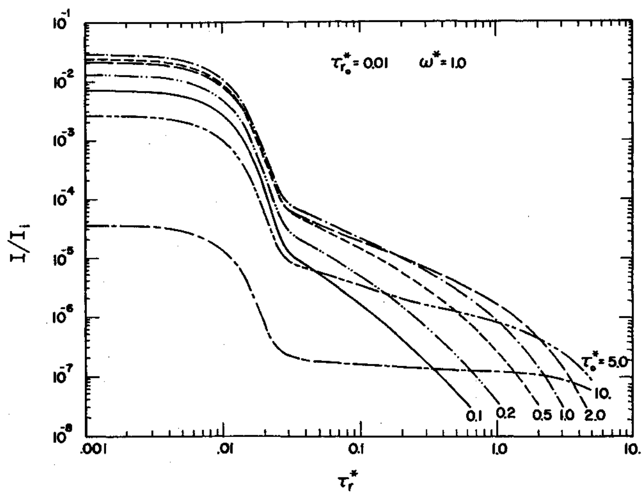

Fig. 3 Theoretical diffuse transmitted intensity vs effective optical radius for several values of effective optical depth.

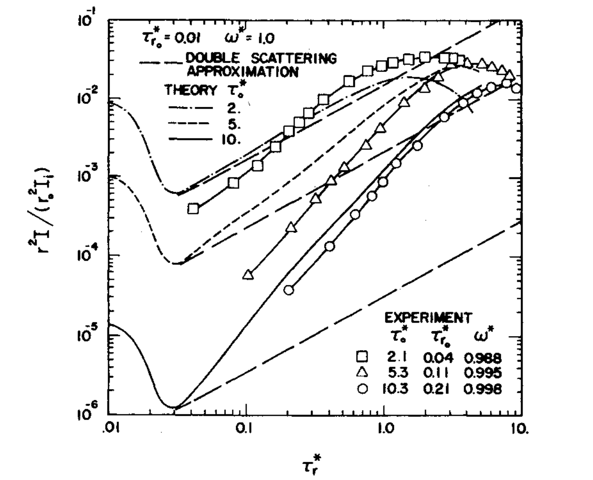

Fig. 4 Experimental and theoretical transmitted intensity from 0.481 μm diameter latex particles vs effective optical radius at effective optical depths of 2, 5, and 10 and $L/R = 0.33$ (symbols are the experimental data, curved lines the theory, and straight, long-dashed lines the double scattering theory approximation).

The Dirac delta function product restricts the incident radiation to the normal direction. The incident radiative flux for this intensity distribution[22] is $I_i \exp[-(\tau_r/\tau_{r_0})^2]$.

Scattering Phase Function

When the wavelength of the radiation is small compared to the scattering particle diameter, the scattering phase function develops a sharp peak in the forward direction. This allows the phase function to be approximated as

$$P(\gamma) = 2g\delta(1-\cos\gamma) + 1 - g \tag{3}$$

in which the asymmetry factor is defined as

$$g = 0.5\int_0^{\pi} \cos\gamma P(\gamma)\,\sin\gamma d\gamma \tag{4}$$

Van de Hulst[29] has stressed that g is the fundamental phase function parameter. Putting this phase function into the equation of radiative transfer yields a modified transfer equation for isotropic scattering, in which the optical coordinates and parameters are modified as

$$\tau_r^* = (1-\omega g)\tau_r,\ \tau_{r_0}^* = (1-\omega g)\tau_{r_0}$$

$$\tau_0^* = (1-\omega g)\tau_0,\ \omega^* = \omega(1-g)/(1-\omega g) \tag{5}$$

in which $\tau_0 = (NC_{\text{sca}} + \kappa)L$ and $\omega = NC_{\text{sca}}/(NC_{\text{sca}} + \kappa)$. Thus, for anisotropic scattering with a peaked phase function, the isotropic solutions for the source function, flux, and intensity can be used if the optical coordinates and albedo are adjusted.[22,23]

Figure 2 shows the normalized scattering phase function for 0.481 μm diameter latex particles as a function of scattering angle. The curve was generated using Mie theory. Latex particles used in the current study have been shown to have scattering phase functions that agree very closely to their theoretical values.[30] The phase function is approximated in the theory as having an δ-function spike at a scattering angle of 0 deg and a value of $1-g$ at all other angles as given by Eq. (3).

Transmission Solution

The intensity transmitted through the media is divided into two parts: the direct transmission of the incident laser radiation and the diffuse, or scattered, radiation. The interface is assumed to perfectly transmit the incident laser radiation. Using Eq. (1) for the direct term, the transmitted flux in the solid angle $\Delta\Omega$ is

$$q(\tau_r^*, \tau_0^*) = I_i \exp[-(\tau_r^*/\tau_{r_0}^*)^2]\ \exp(-\tau_0^*) + I_d(\tau_r^*, \tau_0^*)\Delta\Omega \tag{6}$$

in which the direct term transmission factor is $\exp(-\tau_0^*)$. But, $q = \int I \cos\theta d\Omega = \int I d\Omega$ for $\cos\theta \simeq 1$. Therefore, $I_N(\tau_r^*, \tau_0^*) = q(\tau_r^*, \tau_0^*)/\Delta\Omega$, where $\Delta\Omega$ is an incremental solid angle centered on the normal direction. Thus, from Eq. (6), one has

$$\frac{I_N(\tau_r^*, \tau_0^*)}{I_i} = \frac{\exp[-(\tau_r^*/\tau_{r_0}^*)^2 - \tau_0^*]}{\Delta\Omega} + \frac{I_d(\tau_r^*, \tau_0^*)}{I_i} \tag{7}$$

where τ_0^* is the effective optical thickness of the medium. The diffuse intensity is evaluated perpendicular to the surface (i.e., at $\mu = 1$ and $\phi = 0$). The magnitude of the direct term is very sensitive to the value of the solid angle. Also, the direct term is important only at small τ_r^* values, inside and very close to the laser beam. This paper is concerned with $\tau_r^* > \tau_{r_0}^*$, so the direct term will not be considered further. The general expression for $I_d(\tau_r^*, \tau_0^*)$ involves some very complicated and hard-to-evaluate integrals; consequently, several approximations have been developed.[28]

Approximate Diffuse Transmission Solutions

Crosbie[28] has shown that the general expression for diffuse intensity reduces to

$$I_d(\tau_r^*, \tau_0^*) = \frac{\omega^* \tau_0^* I_i}{4\pi} \exp(-\tau_0^*)\left[\exp\left(-\frac{\tau_r^*}{\tau_{r_0}^*}\right)^2 + \frac{\pi^{3/2}\omega^*\tau_{r_0}^*}{4}\exp\left(-\frac{\tau_r^{*2}}{2\tau_{r_0}^{*\,2}}\right)\bar{I}_0\left(\frac{\tau_r^{*2}}{2\tau_{r_0}^{*\,2}}\right)\right] \tag{8}$$

for single and double scattering and $\tau_r^* > \tau_{r_0}^*$, where $\bar{I}_0[\tau_r^*/(2\tau_{r0}^{*2})]$ is the modified Bessel function. The first term on the right-hand side of Eq. (8) represents single scattering and the second term represents double scattering. Note that Ref. 28 has a misprint in Table 2. In $I_1(r/r_0)$ for the Gaussian case, the function should have (r_0/r), not (r/r_0), in the first set of parentheses.[31]

For $\tau_r^* \gg \tau_{r_0}^*$, or far from the center of the incident laser beam, one obtains a simpler form of Eq. (8) as

$$I_d(\tau_r^*, \tau_0^*) = \left(\frac{\omega^* \tau_{r_0}^*}{4\tau_r^*}\right)^2 \tau_0^* \tau_r^* I_i \exp(-\tau_0^*) \tag{9}$$

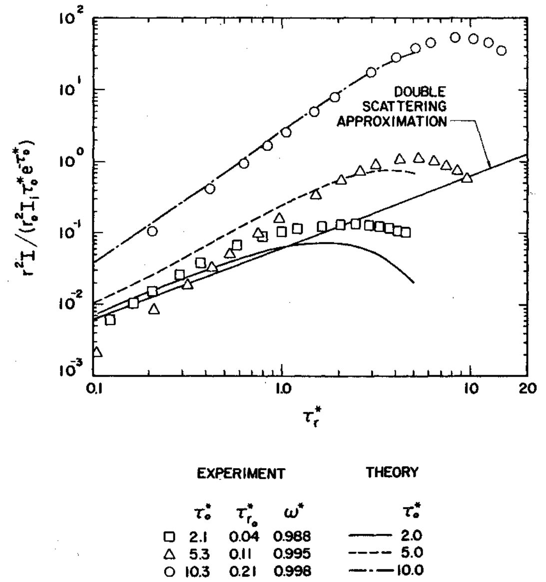

Fig. 5 Experimental and theoretical transmitted intensity from 0.481 μm diameter latex particles vs effective optical radius for $L/R = 0.33$ at effective optical depths of 2, 5, and 10 (double scattering approximation is also shown).

This equation is good in the double scattering τ_r^* regime. Equation (9) can be written in nondimensional form as

$$r^2 I_d/(r_0^2 I_i) = \omega^{*2} \tau_0^* \tau_r^* \exp(-\tau_0^*)/16 \tag{10}$$

This nondimensional form of the diffuse, double scattered, transmitted intensity in independent of $\tau_{r_0}^*$ and linear in τ_r^*. One must be careful in applying Eq. (10) because, in multiple scattering situations at large τ_r^*, the higher-order scattering terms become important, due not only to transmission but also to reflection. This causes $r^2 I/(r_0^2 I_i)$ to decrease with increasing τ_r^* for τ_r^* of the order of 100–1000 times τ_{r0}^*. The linear relationship given by Eq. (10) is accurate over the approximate interval $3\tau_{r0}^* < \tau_r^* > 300\ \tau_{r0}^*$. A nondimensional parameter similar to the left-hand side of Eq. (10) has been effective in correlating reflection from multiple scattering, multidimensional media.[21-23]

An approximate, nondimensional expression that accounts for the optical depth is

$$r^2 I_d \exp(\tau_0^*)/(r_0^2 I_i \tau_0^*) = \omega^{*2} \tau_r^*/16 \tag{11}$$

The nondimensional expressions given by Eqs. (10) and (11) are valid for double scattering. Nondimensional forms of the diffuse part of Eq. (7) similar to those of Eqs. (10) and (11) are used to collapse the transmission data and correlate it with theory.

Theoretical Solution

The exact solution for I_d/I_i has been numerically evaluated for various optical depths. Figure 3 shows the variation of nondimensional transmitted intensity in the normal direction with the optical radius at several optical depths for pure scattering conditions ($\omega^* = 1$) and for $\tau_{r0}^* = 0.01$.

At small values of τ_r^*, single scattering is the major contributor to the transmitted intensity. As τ_r^* increases, the transmitted intensity falls off rapidly due to the decrease in single scattering. This continues until τ_r^* reaches about twice τ_{r0}^* when the curves flatten out, indicating that multiple scattering has become significant. The switch from single to multiple scattering being dominant occurs rather abruptly and indicates the end of the single scattering region. In the multiple scattering region, the rate of decrease of the intensity with radius depends upon the optical depth. For small optical depths, the intensity decreases faster than it does for large optical depths. This occurs because there are fewer scattering events for small optical depth situations; consequently, a larger fraction of the radiation is transmitted in the single scattering region.

Experimental Procedure

The experimental situation is shown schematically in Fig. 1. The source was a 50 mW He-Ne laser ($\lambda_0 = 0.6328$ μm), incident normal to the surface of the scattering media. The scattering media consisted of double distilled water and scattering particles contained in a 30 cm diameter, cylindrical tank. The tank had polycarbonate sides and a glass bottom. A detector with an effective acceptance angle $\bar{\theta}$ of 1.88 deg received the radiation emerging normally from the bottom of the tank as a function of radial position.

Scattering Particles

Polystyrene latex particles were used as the scattering centers. They were spherical and uniform in size. Their diameter was 0.481 μm and their standard deviation 0.0018 μm. The particles had an asymmetry factor of 0.809 and a size parameter ($\pi n d/\lambda_0$) of 3.18; hence, they were anisotropic scatterers.

Data Acquisition Procedure

Specific amounts of latex solution containing 10% by volume latex particles were added to double distilled water to form the scattering media. The volume of the latex solution was determined by weighing it prior to adding it to the double-distilled water. The specific gravity of the latex solution was assumed to be 1.05. The volume of latex particles added to the given volume of water determined the optical characteristics of the medium. No settling or any other discernible variations with time were detected during the data acquisition.

The intensity of the transmitted beam was measured by moving the detector radially outward from the center of the laser beam. Optical fibers made of high-transmission glass were used to transmit the laser light from the detector to the photomultiplier tube (PMT). The PMT converted the light signals to voltages that were quantified using a digital multimeter. Once a data set was taken, additional particles were thoroughly mixed into the water and the process was repeated.

Relation between Experiment and Theory

The intensity of the scattered radiation received by the PMT is inversely proportional to the detector area and the solid angle and directly proportional to the detected power

$$I_{exp} = P_d/(\nu R_0^2 \Delta\Omega) \tag{12}$$

The value of $\Delta\Omega$ is determined from the acceptance angle of the detection system

$$\Delta\Omega = 2\pi(1 - \cos\bar{\theta}) \simeq \pi\bar{\theta}^2 \tag{13}$$

The actual power producing the intensity must be related to the measured voltage. From an independent laboratory measurement, the conversion factor (which is gain dependent) is 2.2×10^{-7} watts/volt. Thus,

$$I_{\text{exp}} = 2.2\times10^{-7}\, V/(\pi^2 R_0^2 \bar{\theta}^2) \tag{14}$$

in which V is the voltage reading in volts.

The intensity distribution of the incident laser beam was Gaussian, so that the laser power incident to the media[23] is $\pi r_0^2 I_i$. Therefore,

$$\frac{I_{\text{exp}}}{I_i} = \frac{2.2\times10^{-7}}{\pi}\frac{V}{P_i}\left(\frac{r_0}{R_0\bar{\theta}}\right)^2 \tag{15}$$

Equation (15) shows that I_{exp} is very sensitive to the values of r_0, R_0, $\bar{\theta}$, and P_i. The sensitivity to r_0 can be eliminated by rearranging

$$\frac{r^2 I_{\text{exp}}}{r_0{}^2 I_i} = \frac{2.2\times10^{-7}}{\pi}\frac{V}{P_i}\left(\frac{r}{R_0\bar{\theta}}\right)^2 \tag{16}$$

The numerical values of r_0, R_0, and $\bar{\theta}$ are known ($r_0 = 0.1$ cm, $R_0 = 0.19$ cm, $\bar{\theta} = 1.88$ deg). The voltages V at various radial positions r and the input power P_i are measured.

Equation (16) can also be written in a nondimensional form to partially account for the optical depth of the scattering medium as

$$\frac{r^2 I_{\text{exp}}}{r_0^2 I_i}\frac{\exp(\tau_0^*)}{\tau_0^*} = \frac{2.2\times10^{-7}}{\pi}\frac{V}{P_i}\left(\frac{r}{R_0\bar{\theta}}\right)^2\frac{\exp(\tau_0^*)}{\tau_0^*} \tag{17}$$

Consequently, one can compare the theoretical solutions given by Eqs. (7), (11), and (12) with the experimental values given by Eqs. (15-17).

The effective values of the optical thickness and the albedo of the scattering medium must be determined. They can be written following Refs. 22 and 23 as

$$\tau_0^* = (1-\omega g)(\eta c+\kappa)L,\ \tau_{r_0}^* = (1-\omega g)(\eta c+\kappa)r_0 \tag{18}$$

and

$$\omega^* = \eta c(1-g)/[\eta c(1-g)+\kappa] \tag{19}$$

in which η is the particle volume concentration and c the scattering cross section divided by the particle volume.

The value of c was determined theoretically from Mie theory for the 0.481 μm diameter latex particles used in the experiment as 22982.50 1/cm; g was evaluated to be 0.809 for a relative index of refraction for polystyrene latex to water of 1.197. The absorption coefficient of distilled water at λ_0 was assumed to be 0.005 1/cm.[22,23] The albedo of the scattering medium was $[\omega = \eta c/(\eta c+\kappa)]$, because the latex particles were nonabsorbing.

Results and Discussion

Data were obtained for effective optical depths of 2–10. The depth of the water in the tank was set at 5, 10, or 15 cm so that the L/R ratio of the scattering medium was ⅓, ⅔, or 1. The maximum effective optical radius of the scattering medium varied from 2 to 30.

Figure 3 shows theoretical results for $\tau_{r_0}^* = 0.01$. The experimental values of $\tau_{r_0}^*$ varied between 0.041 and 0.205. However, comparisons to theory can still be made. Equation (10) shows that the nondimensional, double scattered, transmitted intensity is not a function of $\tau_{r_0}^*$ at large values of r/r_0.

Figure 4 presents $r^2 I/(r_0^2 I_i)$ vs τ_r^* for values from 2 to 10.3 for L/R equal to ⅓. The corresponding exact theoretical solutions and double scattering approximate solutions [see Eq. (10)] for isotropic scattering are also shown. The exact theoretical curves are for $\tau_{r_0}^*$ equal to 0.01. However, the comparison to experimental data at different $\tau_{r_0}^*$ values is acceptable, because Eq. (10) shows that $r^2 I/(r_0^2 I_i)$ should be independent of $\tau_{r_0}^*$ in this τ_r^* range.

The use of effective optical values yields good agreement between theory and experiment. The slope of the data at $\tau_0^* = 2$ is slightly greater than that of the theory. This is due to differences in the phase function model [Eq. (3)] and the phase function of the 0.481 μm particles used in the experiment. The slopes of the theoretical solutions and experimental data agree progressively better as τ_0^* increases, because the large number of scattering events tend to reduce the importance of single and double scattering and hence the details of the scattering phase function. The double scattering approximation is quite good at $\tau_0^* = 2$. However, agreement becomes progressively worse as τ_0^* increases because of higher-order scattering effects.[32]

Figure 5 presents the data of Fig. 4 in the form of $r^2 I_{\text{exp}}/[r_0^2 I_i \tau_0^* \exp(-\tau_0^*)]$ vs τ_r^*. This form of presentation separates the data for each τ_0^* value. The agreement between theory and experiment is quite good at $\tau_0^* = 10$. However, agreement becomes progressively worse as τ_0^* decreases. Also, the agreement generally improves as τ_r^* increases. It was very hard to get repeatable results close to the laser beam center (small τ_r^*) because of the sensitivity of the data to alignment and detector position. The double scattering approximation as given by Eq. (11) is also shown on Fig. 5. This approximation is acceptable only at small τ_0^* values.

Figures 4 and 5 show that the experimental and theoretical results agree well at large optical depths. At optical depths of 5 and 2, the experimental data values are lower than the theoretical solution at small values of τ_r^*; on the other hand, at large τ_r^*, they are higher than the theoretical values. This can be attributed to: 1) the experimental results being for particles that have a strong forward scattering lobe, while the theory uses a simple δ-function plus isotropic scattering phase function model; and 2) the difficulties in obtaining good data close to the beam center.

The agreement between theory and experiment improves as the particle concentration increases, because more scattering events occur. At small particle concentrations, single and double scattering are important; consequently, the details of the phase function are important. At higher scattering particle concentrations, more scattering events occur and they tend to smooth the effects of the phase function angular variation and make the scattered radiation more isotropic. Also, at large τ_0^* and τ_r^*, the intensity varies slowly with radius; whereas, at small τ_0^* and τ_r^*, the intensity has large radial gradients. The detector sees a surface area and averages the intensity over this area. The strong radial gradients cause experimental error at small values of τ_r^*.

The detection system acceptance angle (1.88 deg) and R_0 value (0.19 cm) were too large to obtain accurate data close to the beam center. The effective radius of the beam was 0.10 cm. A data value was taken at the center of the beam. Because of the design of the detection system, the next radial position at which data could be taken was 0.10 cm from the center of the beam. This made it very difficult to obtain experimental data for τ_r^* less than $\tau^*{}_{r_0}$. Also, data taken close to the beam were not very consistent because of its sensitivity to detector position and alignment. Thus, experimental verification of theoretical results was more reliable at larger τ_r^*. The region of small τ_r^* near and in the beam will be investigated in the future.

Figure 6 presents $r^2 I_{\text{exp}}/[r_0^2 I_i \tau_0^* \exp(-\tau_0^*)]$ vs τ_r^* for L/R equal to ⅓, ⅔, and unity for a scattering medium effective optical thickness of 10. The theoretical solution is also shown. It is for $L/R = 0$ because the theoretical solution assumed that the scattering medium had an infinite radius. The experimental results show very good agreement with the theory when they are correlated using the effective optical

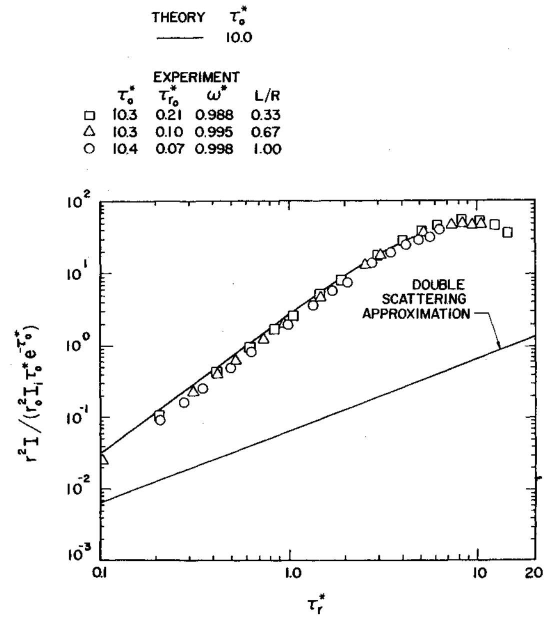

Fig. 6 Experimental and theoretical transmitted intensity from 0.481 μm diameter latex particles vs effective optical radius at $\tau_0^* = 10$ for $L/R = 0.33$, 0.67, and 1.00 (double scattering approximate solution is also shown).

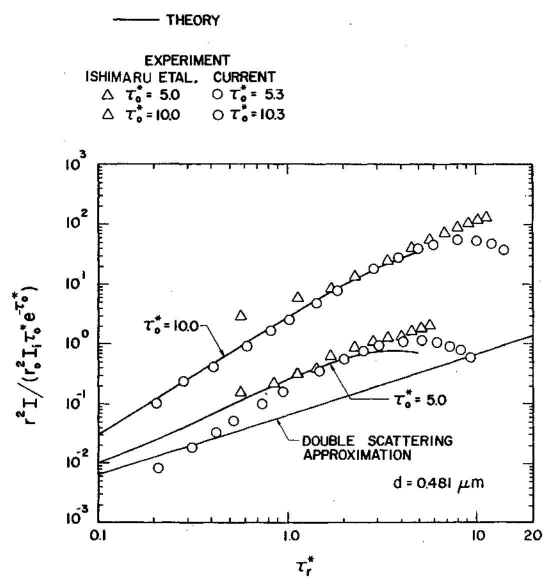

Fig. 7 Comparison between isotropic theory, Ishimaru et al.[26] experiments, and current experiments at $\tau_0^* = 5$ and 10 for 0.481 μm diameter latex particles (double scattering approximation is also shown).

properties and this nondimensional group. The effect of L/R is unimportant for the range of τ_r^*, τ_0^*, and L/R values shown. The double scattering approximation solution is very poor in this case because of the large number of scattering events.

Comparison to Ishimaru et al.

Ishimaru et al.[26] published comparisons between theory and experiment for a problem similar to the one presented herein. However, they were primarily interested in very large optical thicknesses. In their experiment, uniform latex particles were suspended in distilled water in a cylindrical cell 1 cm deep and

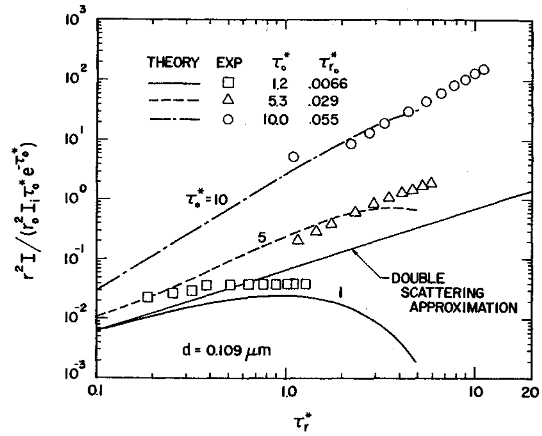

Fig. 8 Comparison between isotropic theory and Ishimaru et al.[26] experiments at $\tau_0^* = 1$, 5, and 10 for 0.109 μm diameter latex particles ($g = 0.090$, $c = 1435$ 1/cm) (double scattering approximation is also shown).

5 cm in diameter. The experiments were done using 0.109, 0.481, 2.02, 5.7, and 11.9 μm diameter polystyrene latex particles. The transmitted radiation normal to the scattering medium was measured at 0.5 mm radial intervals from the center of the incident beam. The detector, which had a field of view of 1.79 deg, was mounted on a computer controlled stage having a range of movement of 1 cm. A He-Ne laser at a wavelength of 0.6328 μm, an output of 5 mW, and a beam diameter of 1.1 mm was used as the light source. The measured radial variation of the transmission was presented in decibels vs radial distance r.

Figure 7 compares $r^2 I_{\exp}/[r_2^2 I_i \tau_0^* \exp(-\tau_0^*)]$ vs τ_r^* for Ishimaru's experiments, the current experiments, the current theory, and the double scattering approximation for effective optical depths of 5 and 10 for the 0.481 μm diameter particles. In order to compare Ishimaru's results to our results, one of Ishimaru's data points was adjusted to agree and this adjustment factor was used for all the data. Figure 8 shows the comparison of Ishimaru's experiments and the current theory for effective optical depths of 1, 5, and 10 for 0.109 μm diameter particles. The experimental results of Ishimaru et al. give a completely independent check of the current experimental and theoretical analysis and reinforce our confidence in the current experimental and theoretical results.

Summary and Conclusions

Transmission of He-Ne laser radiation through two-dimensional anisotropic scattering media in the normal direction was measured and compared to theory. The incident laser beam was normal to the surface of the scattering media. The transmitted intensity was measured as a function of optical thickness of the scattering media and optical radius from the center of the laser beam.

The scattering medium was composed of distilled water and uniform-sized 0.481 μm diameter polystyrene latex spheres. This corresponds to a particle size parameter of 3.18 and an asymmetry factor of 0.809.

The experimental transmission results compared well with theoretical scattering results when they were correlated using effective optical parameters. The results were independent of the ratio of the radius to the depth of the scattering medium when they were plotted using the nondimensional parameter $(r^2 I)/[r_0^2 I_i \tau_0^* \exp(-\tau_0^*)]$. This nondimensional parameter collapsed the data for each value of τ_0^* to a single curve independent of the physical size of the scattering medium. Accurate experimental data were not obtained near the beam because of the large detection system acceptance angle and the sensitivity of the measured radiation to detector position. At small optical depths and optical radii, the experimental results showed a smaller transmission than the theory predicted. This was attributed to the large acceptance angle and detection area of the detector and differences between the model and exact anisotropic scattering phase function.

The double scattering approximation is reasonable at small effective optical depths (τ_0^* less than about 2) for $0.1 < \tau_r^* < 5$. However, as τ_0^* increases, the approximation becomes progressively more inaccurate.

In conclusion: 1) the validity of the general theoretical solution has been shown by comparison with the experimental results; 2) the results are not sensitive to the ratio of depth to radius of the scattering media (in the range between 0.33 and 1.0) for optical depths from 2 to 10; 3) the effects of anisotropic multiple scattering are important; and 4) the effect of the angular details of the scattering phase function tend to be washed out for large optical depths due to multiple scattering.

In the future, this research should be extended to include laser beam incidence at angles other than normal to the surface of media. This will make the problem three-dimensional. Also, the effects of transmission at small τ_r^* where the direct term is important, polarization, and L/R values greater than unity make the edge effects important. These effects need to be investigated.

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